Review

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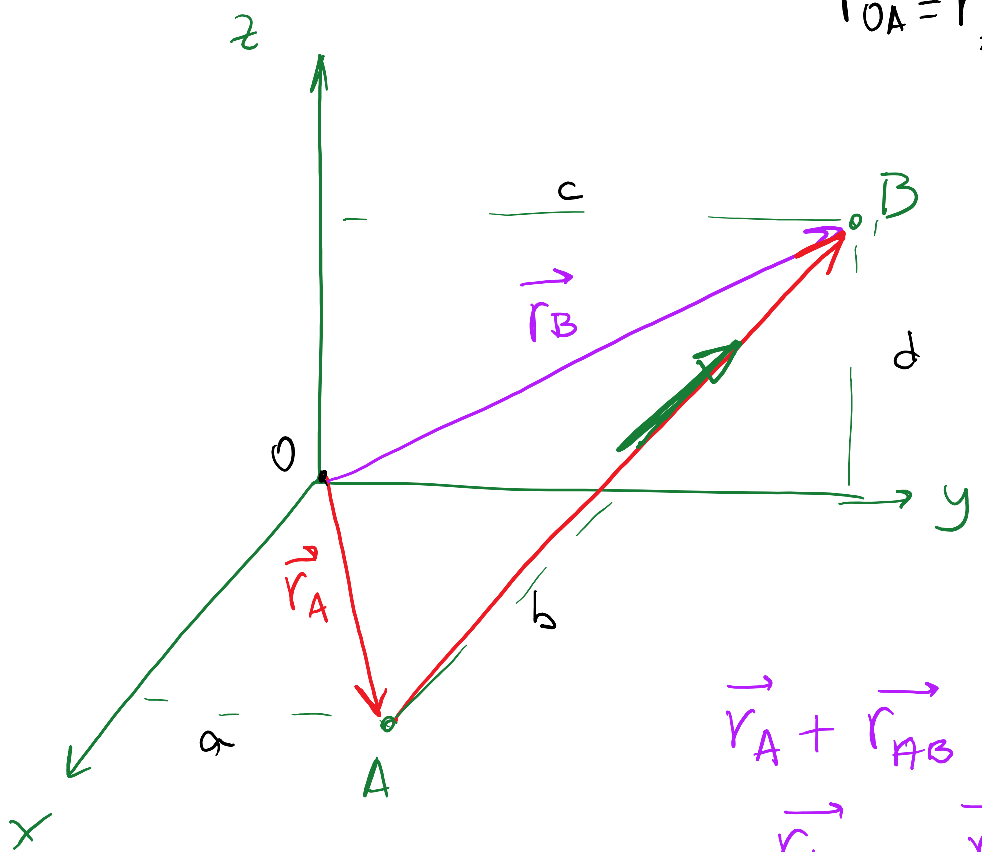


$$\vec{r}_{OA} = \vec{r}_A = \langle b, a, 0 \rangle$$

$$\vec{r}_B = \langle 0, c, d \rangle$$

$$\vec{u}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|}$$

$$\vec{r}_A + \vec{r}_{AB} = \vec{r}_B$$

$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$$

$$\vec{u}_{AB} = \frac{(\vec{r}_B - \vec{r}_A)}{|\vec{r}_B - \vec{r}_A)}$$

$$\vec{a} \cdot \vec{b} =$$

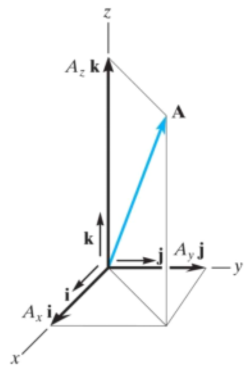
$$\langle a_x, a_y, a_z \rangle \cdot \langle b_x, b_y, b_z \rangle$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z \Longrightarrow \text{SCALAR!!}$$

Note that: $\vec{A}\cdot\hat{i} = A_x$

$\vec{A}\cdot\hat{j} = A_y$

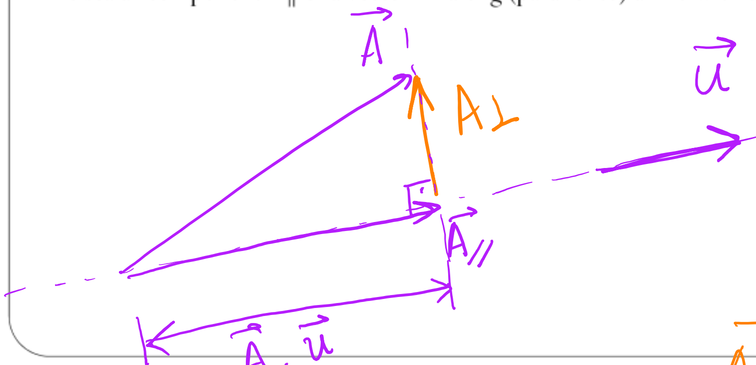
$\vec{A}\cdot\hat{k} = A_z$



$\vec{A} = \langle A_x, A_y, A_z \rangle$

Projections

The scalar component $A_{\parallel}$ of a vector $\boldsymbol{A}$ along (parallel to) a line with unit vector $\boldsymbol{u}$ is given by:



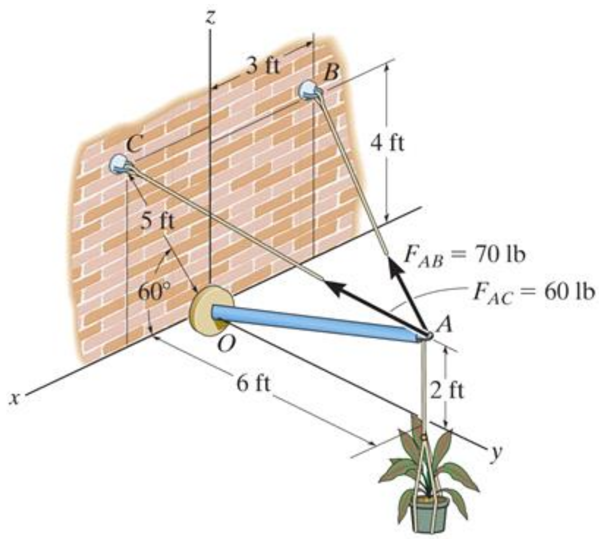
$\vec{A}\cdot\vec{u}$

$\vec{A}_{\parallel} = (\vec{A}\cdot\vec{u})\,\vec{u}$

$\vec{A}_{\perp} = \;?$

$\vec{A}_{\parallel} + \vec{A}_{\perp} = \vec{A}$

$\boxed{A_{\perp} = \vec{A} - \vec{A}_{\parallel}}$



$$\vec{F}_{AC} \cdot \frac{\vec{r}_{AO}}{|\vec{r}_{AO}|} \quad = \vec{u}_{AO}$$

$\vec{r}_C = \langle 5\cos 60°, 0, 5\sin 60° \rangle$ ft

$\vec{r}_A = \langle 0, 6, 2 \rangle$ ft

$|\vec{r}_A| = \sqrt{6^2 + 2^2}$ ft

$$\vec{F}_{AC} = F_{AC}\,\vec{u}_{AC} = 60\,\frac{\vec{r}_{AC}}{|\vec{r}_{AC}|} \quad ✓$$

$$\vec{r}_{AC} = \vec{r}_C - \vec{r}_A = \langle 5\cos 60°, -6, 5\sin 60° - 2 \rangle \text{ ft}$$

$$|\vec{r}_{AC}| = \sqrt{(5\cos 60)^2 + 6^2 + (5\sin 60 - 2)^2} = \underline{\qquad\quad} \text{ ft}$$

$$\vec{u}_{AC} = \frac{\vec{r}_{AC}}{|\vec{r}_{AC}|} \implies$$

$$\vec{u}_{AO} = \frac{\vec{r}_{AO}}{|\vec{r}_{AO}|} = \frac{\cancel{\vec{r}_O}^{\,0} - \vec{r}_A}{|\vec{r}_O - \vec{r}_A|} = \frac{-\vec{r}_A}{|\vec{r}_A|}$$

$$\vec{F}_{AC} \cdot \vec{u}_{AO} = \underline{\text{scalar}}$$

$F_{AC} \cdot u_{AO} = \underline{\text{scalar}}$

$$\boxed{\left(\vec{F}_{AC}\right)_{\text{along line } AO} = \left(\vec{F}_{AC} \cdot \vec{u}_{AO}\right) \vec{u}_{AO}}$$

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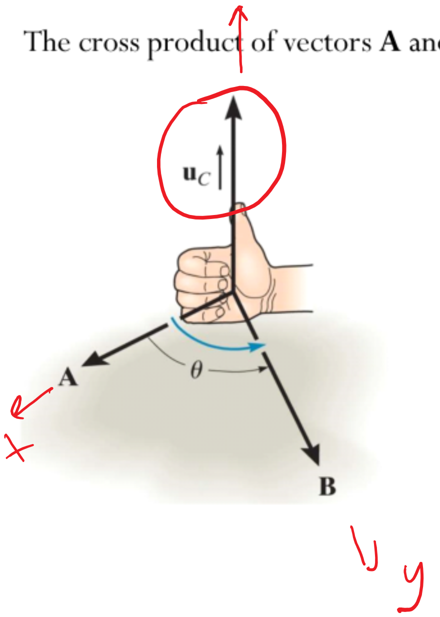
Cross (or vector) product

The cross product of vectors **A** and **B** yields the vector **C**, which is written

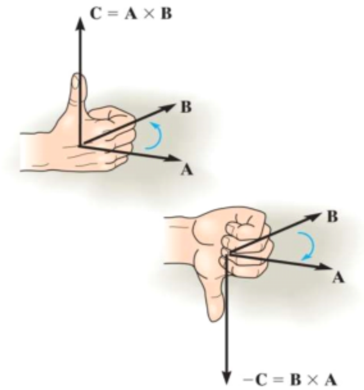


$$\boxed{\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}}$$

$$|\vec{C}| = |\vec{A}||\vec{B}| \sin\theta$$

$$\vec{C} = |\vec{C}|\,\vec{u}_c$$

Cross (or vector) product



Laws of operation:

$$\boldsymbol{A} \times \boldsymbol{B} = -\boldsymbol{B} \times \boldsymbol{A}$$

$$\alpha(\boldsymbol{A} \times \boldsymbol{B}) = (\alpha \boldsymbol{A}) \times \boldsymbol{B} = \boldsymbol{A} \times (\alpha \boldsymbol{B}) = (\boldsymbol{A} \times \boldsymbol{B})\alpha$$

$$\boldsymbol{A} \times (\boldsymbol{B} + \boldsymbol{D}) = \boldsymbol{A} \times \boldsymbol{B} + \boldsymbol{A} \times \boldsymbol{D}$$

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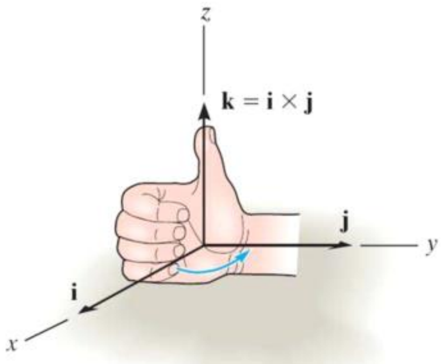
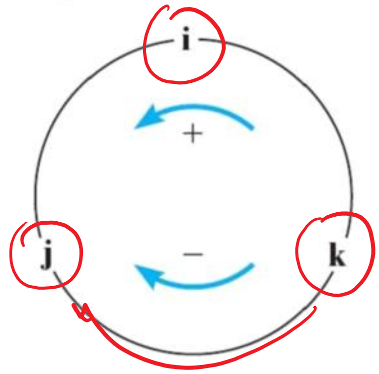
Cross (or vector) product

The right-hand rule is a useful tool for determining the direction of the vector resulting from a cross product. Note that a vector crossed into itself is zero, e.g., $\boldsymbol{i} \times \boldsymbol{i} = 0$





Considering the cross product in Cartesian coordinates

$$\vec{A} \times \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

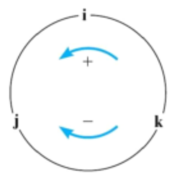
$$+ A_x B_y \hat{k} - A_x B_z \hat{j} - A_y B_x \hat{k} + A_y B_z \hat{i}$$

$$+ A_z B_x \hat{j} - A_z B_y \hat{i}$$

Cross (or vector) product

Also, the cross product can be written as a determinant.

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$



$\vec{M} = \vec{r} \times \vec{F}$

$\vec{C} = \vec{A} \times \vec{B}$

Each component can be determined using 2 × 2 determinants.

$$+\begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} - \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} + \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{C} = +(A_y B_z - A_z B_y)\,\hat{i} - (A_x B_z - A_z B_x)\,\hat{j} + (A_x B_y - B_x A_y)\,\hat{k}$$

$\vec{C} = \vec{A} \times \vec{B}$

Equilibrium of a particle

According to Newton's first law of motion , a particle will be in **equilibrium** (that is, it will remain at rest or continue to move with constant velocity) if and only if

$$\sum \boldsymbol{F} = \mathbf{0}$$

where $\sum \boldsymbol{F} = \mathbf{0}$ is the resultant force vector of all forces acting on a particle.

In three dimensions, equilibrium requires:

$$\sum \vec{F} = \underbrace{\sum F_x}_{=0} \hat{i} + \underbrace{\sum F_y}_{=0} \hat{j} + \underbrace{\sum F_z}_{=0} \hat{k} = \vec{0}$$

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum F_z = 0$$

Coplanar forces: if all forces are acting in a single plane, such as the "xy" plane, then the equilibrium condition becomes

$$\sum F_x = 0$$
$$\sum F_y = 0$$

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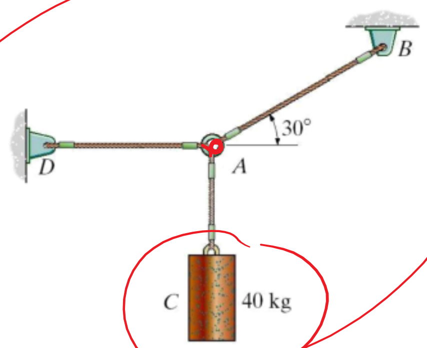


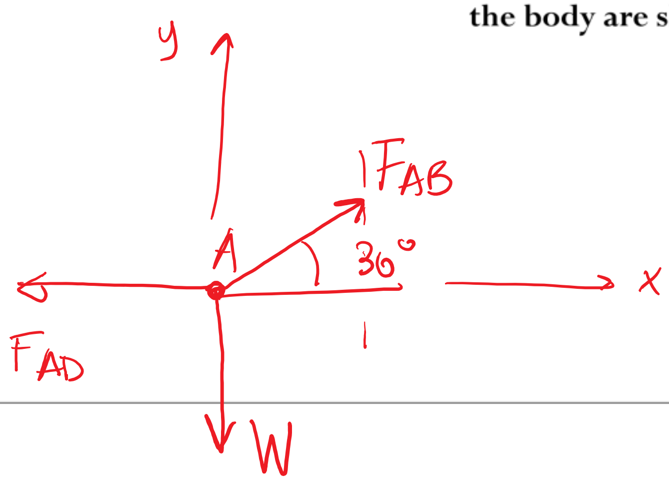
This is an example of a 2-D or coplanar force system.

If the whole assembly is in equilibrium, then particle A is also in equilibrium.

To determine the tensions in the cables for a given weight of cylinder, you need to learn how to draw a free body diagram and apply the equations of equilibrium.

Free body diagram

Drawing of a body, or part of a body, on which all the forces acting on the body are shown.



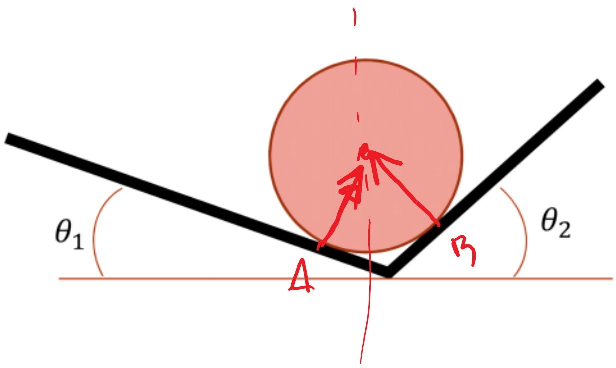
$$\Sigma F_x = 0 \qquad F_{AB}\cos 30^\circ - F_{AD} = 0$$

$$\Sigma F_y = 0 \qquad F_{AB}\sin 30 - W = 0 \qquad g = 9.81\ m/s^2$$

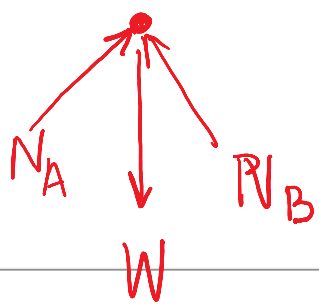
$$F_{AB} = \frac{W}{\sin 30^\circ} \quad //$$

$$F_{AD} = F_{AB}\cos 30^\circ = \frac{\overset{mg}{W}}{\sin 30^\circ}\cos 30^\circ \quad //$$

Equilibrium of a particle (cont.)



Contact force in smooth surface:
Consider now the uniform sphere of weight W, supported by smooth (frictionless) surfaces. Because the contact surfaces are smooth, the forces exerted on the sphere by the planes must be perpendicular to the surface.



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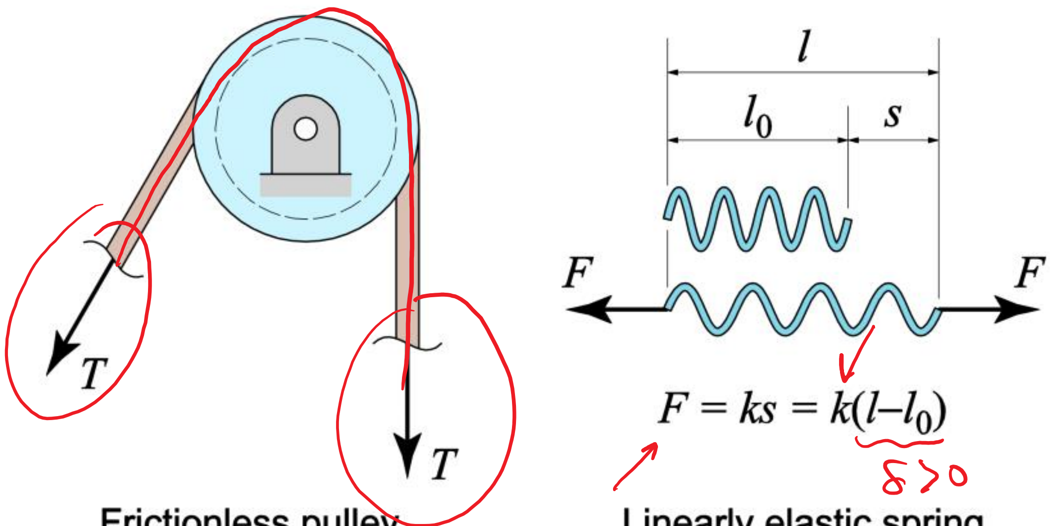
Idealizations

Pulleys are (usually) regarded as frictionless; then the tension in a rope or cord around the pulley is the same on either side. Springs are (usually) regarded as linearly elastic; then the tension is proportional to the *change* in length *s*.



Frictionless pulley

$F = ks = k\underbrace{(l - l_0)}_{\delta > 0}$

Linearly elastic spring

$\delta = l_f - l_i$

$\delta < 0$

$F = k\delta$

$F < 0$

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